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Heat and mass transfer in unsteady-state diffusion combustion of a condensed fuel surface

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Abstract—A solution is obtained for the unsteady-state one-dimensional problem of condensed fuel surface combustion in an oxidant atmosphere. As with the majority of diffusion combustion problems, the chemical reaction rate in the flame zone is supposed to be large compared with the rate of diffusion. An analytical solution is obtained for the system of nonstationary one-dimensional equations describing the flow of a heat conducting, reacting mixture of mutually diffusing gases. A comparison is made between the computed results and experimental data on the diffusion combustion of gasoline in fine tubes.

1. INTRODUCTION

IT HAS BEEN established experimentally [1–5] that during combustion of the surface of liquid and solid fuels in an oxidant atmosphere, the chemical reaction is localized within a narrow zone termed the diffusion flame surface. In solving the problems of diffusion combustion [6–9] it is supposed that in the flame zone the chemical reaction rate is large as compared with the rate of diffusion and the reactants are unable to penetrate deep through the flame not entering into reaction. The main limiting factor in the determination of the rate of material combustion is not the kinetics of chemical reactions, but rather the rates of combustible evaporation and supply of reactants into the reaction zone by diffusion. Therefore, in the study of diffusion combustion of great importance is the insight into the processes of interphase heat and mass transfer.

The majority of works dealing with such problems consider steady diffusion flames which are stabilized in an oxidant flow [5–9] or which, in the absence of flow, develop above the surface of a burning spherical particle [10]. However, on ignition of unmixed systems the diffusion flame surface originates near the phase interface and recedes from it as time proceeds. This behaviour of the flame surface can be illustrated by solving the unsteady one-dimensional problem of plane fuel surface diffusion combustion in an oxidant atmosphere in the absence of external body forces (Fig. 1).

A problem will be considered on the development of combustion in unmixed systems when, prior to ignition, the density of combustible vapours above the condensed phase surface is close to zero and the flame can originate only close to the surface. Problems, in which a combustible had an ample time to evaporate and by the moment of ignition the vapour density above the surface has become high, are excluded from the analysis, since at the initial stage of flame propagation these problems are close in physical nature to those of homogeneous combustion.

2. MATHEMATICAL STATEMENT OF THE PROBLEM

A system of equations for a one-dimensional unsteady flow of a gaseous mixture with variable composition and chemical reactions reads:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \tag{1}$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} = \frac{\partial}{\partial x} \tilde{\mu} \frac{\partial v}{\partial x} - \frac{\partial p}{\partial x} \tag{2}$$

$$\rho \frac{\partial Y_i}{\partial t} + \rho v \frac{\partial Y_i}{\partial x} = \frac{\partial}{\partial x} \rho D \frac{\partial Y_i}{\partial x} + \omega_i; \quad i = 1, \ldots, N \tag{3}$$

$$\rho \frac{\partial H}{\partial t} + \rho v \frac{\partial H}{\partial x} = \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \tilde{\mu} v \frac{\partial v}{\partial x} + \frac{\partial}{\partial x} \rho D \sum_{i=1}^{N} h_i \frac{\partial Y_i}{\partial x} \tag{4}$$

$$p = \rho R T \sum_{i=1}^{N} \frac{Y_i}{m_i}, \tag{5}$$

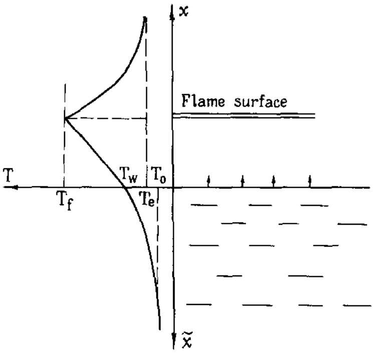

FIG. 1. Temperature distribution in a gas and condensed phase in diffusion combustion.

NOMENCLATURE

D	mean binary diffusion coefficient
c_{pi}	specific heat at constant pressure for ith species
h_i°	specific enthalpy of ith species formation (chemical energy of ith species)
h_L	specific heat of phase change
m_i	molar mass of ith species
p	pressure
R	universal gas constant
T	gas temperature
v	gas velocity
Y_i	mass concentration of ith species.

Greek symbols

λ	mean thermal conductivity
μ	dynamic viscosity
$\tilde{\mu}$	$= 4/3\,\mu$
ν_i', ν_i''	stoichiometric coefficients of ith species before and after reaction
ρ	gas density
ω_i	mass rate of ith species formation as a result of chemical reaction.

Subscripts

$i = 1$	refers to oxidant
$i = N$	refers to fuel
$i = 2,\ldots,N-1$	refers to reaction products and inert components.

where

$$H = \hat{c}_p T + \frac{v^2}{2} + \sum_{i=1}^{N} Y_i h_i^\circ;$$

$$\hat{c}_p = \sum_{i=1}^{N} Y_i c_{pi}; \quad h_i = c_{pi} T + h_i^\circ.$$

As a follow-up to ref. [10], the gas velocities caused by combustion are assumed to be insignificant so that the effects due to friction and dynamic pressure can be neglected. Then, equation (2) can be replaced by the isobaricity condition (p = const.) usually used in the solution of diffusion combustion problems [5–10, 12, 13].

Multiply equation (3) by h_i°, sum over i and subtract from equation (4). Add equation (4) to equation (3), which has been multiplied by $\hat{c}_p T$. Assuming the Lewis number ($Le = \lambda/\rho D \hat{c}_p$) to be close to unity, one obtains a system of equations equivalent to the original one:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \tag{6}$$

$$p = \text{const.} \tag{7}$$

$$\frac{\partial \rho Y_i}{\partial t} + \frac{\partial \rho v Y_i}{\partial x} = \frac{\partial}{\partial x} \rho D \frac{\partial Y_i}{\partial x} + \omega_i \tag{8}$$

$$\frac{\partial \rho \hat{c}_p T}{\partial t} + \frac{\partial \rho v \hat{c}_p T}{\partial x} = \frac{\partial}{\partial x} \frac{\lambda}{\hat{c}_p} \frac{\partial \hat{c}_p T}{\partial x} - \sum_{i=1}^{N} \omega_i h_i^\circ. \tag{9}$$

Taking into account the fact that for the proceeding chemical reaction the relation $\omega_i = \omega m_i(\nu_i'' - \nu_i')$ [9, 10] is valid, the above system yields the following equations to determine the pair functions β_i:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \tag{10}$$

$$\frac{\partial \rho \beta_i}{\partial t} + \frac{\partial \rho v \beta_i}{\partial x} = \frac{\partial}{\partial x} \frac{\lambda}{\hat{c}_p} \frac{\partial \beta_i}{\partial x}; \quad i = 2,\ldots,N,T,C, \tag{11}$$

where

$$\beta_i = \frac{Y_i}{m_i(\nu_i'' - \nu_i')} - \frac{Y_1}{m_1(\nu_1'' - \nu_1')}; \quad i = 2,\ldots,N$$

$$\beta_T = \frac{\hat{c}_p T}{m_N(\nu_N' - \nu_N'')\,\Delta H} - \frac{Y_1}{m_1(\nu_1'' - \nu_1')}$$

$$\Delta H = \frac{\sum_{i=1}^{N} m_i h_i^\circ(\nu_i' - \nu_i'')}{m_N(\nu_N' - \nu_N'')}$$

$$\beta_C = \frac{\hat{c}_p}{m_1(\nu_1'' - \nu_1')\,\Delta c_p} - \frac{Y_1}{m_1(\nu_1'' - \nu_1')}$$

$$\Delta c_p = \frac{\sum_{i=1}^{N} m_i c_{pi}(\nu_i'' - \nu_i')}{m_1(\nu_1'' - \nu_1')}.$$

In the solution of the unsteady problem the fuel cannot be assumed as uniformly preheated. The energy equation in a fuel layer in a coordinate system moving with the phase interface (Fig. 1) has the form

$$\frac{\partial T}{\partial t} + v_f \frac{\partial T}{\partial x} = \left(\frac{\lambda}{\rho c}\right)_f \frac{\partial^2 T}{\partial x^2}, \tag{12}$$

where $v_f(t)$ is the velocity of fuel motion relative to the phase interface.

The boundary conditions for system (10)–(12) are written in the form:

(1) at the outer boundary in the gas ($x \to +\infty$)

$$T = T_e, \quad Y_1 = Y_{1e}, \quad Y_{Ne} = 0; \tag{13}$$

(2) at the outer boundary in the liquid

$$T = T_0; \tag{14}$$

(3) at the phase interface ($x = 0$) [12, 13]

$$(\rho v)_W = -\rho_f v_f, \tag{15}$$

$$(\rho v)_W Y_{iW} - (\rho D)_W \frac{\partial Y_i}{\partial x} = 0, \quad i = 1,\ldots,N-1;$$

$$(\rho v)_W(Y_{NW}-1)-(\rho D)_W\frac{\partial Y_N}{\partial x}=0; \qquad (16)$$

$$-\lambda_f\left(\frac{\partial T}{\partial x}\right)_{fW}=\lambda\left(\frac{\partial T}{\partial x}\right)_W-(\rho v)_W h_L. \qquad (17)$$

Boundary conditions (13)–(17) are supplemented with the condition of equilibrium evaporation [7, 14] and with the assumption of the diffusion mode of combustion, i.e. an oxidant is absent ($i = 1$) under the flame surface and a fuel is absent ($i = N$) above the flame surface.

3. SOLUTION OF THE SYSTEM OF EQUATIONS

The system of equations (11) admits the first integrals

$$\beta_i=(\beta_{ie}-\beta_{iW})\beta+\beta_{iW}, \quad i=2,\ldots,N,T,C, \qquad (18)$$

where the function β is also the solution of equation (11) subject to the boundary conditions

$$\beta=0 \text{ at } x=0 \quad\text{and}\quad \beta=1 \text{ when } x\to+\infty.$$

To obtain the solution of system (10), (11), introduce a new independent similarity variable $\xi = x/\sqrt{2t}$ and dependent variables

$$\varphi(\xi)=\rho/\rho_e; \quad \rho v=\rho_e[f(\xi)/\sqrt{2t}]; \quad \beta=\beta(\xi). \qquad (19)$$

The system of equations (10) and (11) takes the form

$$-\varphi'\xi+f'=0 \qquad (20)$$

$$-\xi\varphi\beta'+f\beta'=\frac{1}{\rho_e Pr}(\mu\beta''+\beta'\mu'), \qquad (21)$$

where the prime means differentiation with respect to $\xi(f' = \mathrm{d}f/\mathrm{d}\xi)$, $Pr = \mu c_p/\lambda =$ const. is the Prandtl number.

It will be assumed, as is usually done when homobaric processes are considered [10, 11, 12], that the condition $\rho\mu =$ const. is fulfilled throughout the region. From this condition it follows that

$$\mu'\varphi+\varphi'\mu=0; \quad \mu=\mu_e/\varphi. \qquad (22)$$

The substitution of relations (22) into relation (21) yields

$$a^2\left(\frac{\beta''}{\varphi}-\frac{\beta'\varphi'}{\varphi^2}\right)=\beta'(f-\xi\varphi) \qquad (23)$$

where $a^2 = \mu_e/\rho_e Pr$.

Similarly, by introducing new variables for the condensed phase ($\tilde{x} > 0$)

$$\xi=\tilde{x}/\sqrt{2t}; \quad \psi(\xi)=T/T_0 \qquad (24)$$

and taking into account boundary condition (15)

$$(\rho v)_W=-\rho_f v_f=\rho_e f(0)/\sqrt{2t},$$

one may reduce the energy equation (11) to the form

$$b^2\psi''(\xi)+\psi'(\xi)(\xi+k)=0 \qquad (25)$$

where

$$k=\frac{\rho_e}{\rho_f}f(0); \quad b^2=\frac{\lambda_f}{\rho_f c_f}.$$

The resulting system of three equations (20), (23) and (25) contains four unknown functions: $f(\xi)$, $\varphi(\xi)$, $\beta(\xi)$, and $\psi(\xi)$. To bring about closure of the system, it is necessary to use one other equation, i.e. equation (2), which in the present case has degenerated into the homobaricity condition (7), whence, with the aid of the state equation for a multi-component mixture

$$p_e=\rho RT\sum_{i=1}^{N}\frac{Y_i}{m_i}$$

the following relations will be obtained

$$\frac{p_e}{\rho_e R}=\varphi(\xi)\left[\beta(\xi)\sum_{i=2}^{N}(v_i''-v_i')(\beta_{ie}-\beta_{iW})+\sum_{i=2}^{N}\beta_{iW}(v_i''-v_i')\right]\frac{\beta(\xi)(\beta_{Te}-\beta_{TW})+\beta_{TW}}{\beta(\xi)(\beta_{Ce}-\beta_{CW})+\beta_{CW}} \qquad (26a)$$

for $\xi < \xi_*$, i.e. below the flame surface,

$$\frac{p_e}{\rho_e R}=\varphi(\xi)\left[\sum_{i=2}^{N}(v_i''-v_i')\beta_i(\xi)-\beta_N(\xi)\times\sum_{i=2}^{N}\frac{\Phi_i}{m_i}\right]\frac{\beta_T(\xi)+\beta_N(\xi)\Delta H}{\beta_C(\xi)-\beta_N(\xi)\Delta c_p} \qquad (26b)$$

for $\xi > \xi_*$, i.e. above the flame surface, where

$$\Phi_i=\frac{m_i(v_i''-v_i')}{m_1(v_1''-v_1')}.$$

The first integrals of the system of equations in the gas (18) and boundary conditions (15)–(17) yield the relations to determine the concentrations and heat fluxes at the phase interface:

$$\beta_{iW}=\frac{\beta_{ie}}{1+\{[\rho_e f(0)]/[(\rho D)_W\beta'(0)]\}}; \quad i=2,\ldots,N-1 \qquad (27)$$

$$\beta_{NW}=\frac{\beta_{Ne}+\{[\rho_e f(0)]/[(\rho D)_W\beta'(0)]\}}{1+\{[\rho_e f(0)]/[(\rho D)_W\beta'(0)]\}}; \qquad (28)$$

$$-\lambda_f T_0\psi'(0)=Le\,\hat{c}_{pW}(\rho D)_W\beta'(0)\left[\frac{\beta_{Te}-\beta_{TW}}{\beta_{CW}}-\frac{\beta_{Ce}-\beta_{CW}}{\beta_{CW}^2}\beta_{TW}-\frac{\rho_e f(0)h_L}{Le\,\hat{c}_{pW}(\rho D)_W\beta'(0)}\right]. \qquad (29)$$

By introducing the dimensionless mass transfer parameter $B_D = [\rho_e f(0)]/[(\rho D)_W\beta'(0)]$ and using the definition of β_i, the following relations can be obtained from relations (27)–(29):

$$Y_{iW}=\frac{Y_{ie}-\Phi_i Y_{1e}}{1+B_D}; \quad i=2,\ldots,N-1; \qquad (30)$$

$$Y_{NW}=\frac{B_D-\Phi_N Y_{1e}}{1+B_D};$$

$$-\lambda_f T_0 \psi'(0) = (\rho D)_W \beta'(0) \times [\hat{c}_{pe}(T_e - T_W) + Y_{1e} Q_{pW} - B_D h_L], \quad (31)$$

where

$$Q_{pW} = T_W \Delta c_p + \Phi_N \Delta H; \quad Le = 1.$$

Determine the partial pressure of combustible vapours at the phase interface by the formula

$$p_{NW} = Y_{NW} p_e \frac{m_W}{m_N}, \quad \text{where} \quad \frac{1}{m_W} = \sum_{i=1}^{N} \frac{Y_{iW}}{m_i}. \quad (32)$$

The mass transfer parameter B_D will be determined, similarly to works [8, 9, 14], by substituting relations (30) into equation (32) and making use of the Clapeyron–Clausius modified equilibrium evaporation condition [7, 14]:

$$B_D = Y_{1e}\Phi_N + \sum_{i=2}^{N-1} (Y_{ie} - Y_{1e}\Phi_i) \frac{m_N}{m_i} \left\{ \left(\frac{T_{ke}}{T_W} \right)^{(\Delta c m_N)/R} \times \exp\left[\frac{h_{LO} m_N}{R} \left(\frac{1}{T_{ke}} - \frac{1}{T_W} \right) \right] - 1 \right\}^{-1}. \quad (33)$$

At $\xi = 0$, equations (26) give an additional boundary condition

$$\frac{p_e m_N}{\rho_e R} = \frac{\varphi(0) T_W}{1 + B_D} \left[\sum_{i=2}^{N} \frac{m_N}{m_i} (Y_{ie} - \Phi_i Y_{1e}) + B_D \right]. \quad (34)$$

Equations (20), (23), (25), (26) with their boundary conditions and relations (31), (33), (34) constitute a closed system of three differential and four algebraic equations for the determination of the unknown functions $\varphi(\xi), f(\xi), \beta(\xi), \psi(\xi)$ and unknown parameters B_D, T_W, $\varphi(0)$. The values of the quantities $\beta'(0)$ and $\psi'(0)$ in relations (31) are determined from the solution of corresponding equations.

The solution of equation (25) subject to the boundary conditions

$$\psi(0) = \frac{T_W}{T_0}, \quad \psi(\infty) = 1$$

has the form [12]:

$$\psi(\xi) = 1 + \frac{T_W - T_0}{T_0} \cdot \frac{\operatorname{erfc}[(\xi + k)/b\sqrt{2}]}{\operatorname{erfc}(k/b\sqrt{2})}, \quad (35)$$

where

$$\operatorname{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-z^2} \mathrm{d}z.$$

Equation (35) yields

$$\psi'(0) = -\frac{T_W - T_0}{T_0} \frac{\sqrt{2}}{b\sqrt{\pi}} e^{-k^2/2b^2} \times [\operatorname{erfc}(k/b\sqrt{2})]^{-1}. \quad (36)$$

The LHS of equation (23) contains the total differential

$$a^2 \left(\frac{\beta'}{\varphi} \right)' = \beta'(f - \xi\varphi). \quad (37)$$

Note that equation (20) gives

$$(f - \xi\varphi)' = f' - \xi\varphi' - \varphi = -\varphi \quad (38)$$

allowing equation (37) to be rewritten in the form

$$a^2 \left(\frac{\beta'}{\varphi} \right)' = -\frac{\beta'}{\varphi} (f - \xi\varphi)'(f - \xi\varphi) \quad (39a)$$

or

$$a^2 z' = -zyy',$$

where

$$z = \frac{\beta'}{\varphi}; \quad y = f - \xi\varphi. \quad (39b)$$

The integration of equation (39) yields

$$\beta'(\xi) = C\varphi(\xi) \exp\{-[f(\xi) - \xi\varphi(\xi)]^2/2a^2\} \quad (40)$$

i.e.

$$\beta'(\xi) = C\varphi(\xi) e^{-(y^2/2a^2)}.$$

One obtains $\mathrm{d}y/\mathrm{d}\xi = -\varphi$ from condition (38) and integrates it on having substituted into equation (40)

$$\beta = -C \int_{f(0)}^{y} e^{-(y^2/2a^2)} \mathrm{d}y. \quad (41)$$

By determining the constant C from the condition at the infinity, obtain one other first integral of system (6)–(9):

$$\beta = \frac{\operatorname{erf}[f(0)/a\sqrt{2}] - \operatorname{erf}[(f - \xi\varphi)/a\sqrt{2}]}{1 + \operatorname{erf}[f(0)/a\sqrt{2}]}. \quad (42)$$

The value of C in relation (41) is determined from the formula

$$C = \frac{\sqrt{2}}{a\sqrt{\pi} \cdot \{1 + \operatorname{erf}[f(0)/a\sqrt{2}]\}}.$$

By substituting solution (42) into relation (26) express the function φ as

$$\varphi = g(y). \quad (43)$$

Then, from equations (38) and (43) the following ordinary differential equation is obtained

$$\frac{\mathrm{d}y}{\mathrm{d}\xi} = -g(y); \quad y(0) = f(0), \quad (44)$$

which can be integrated numerically when the initial condition $f(0)$ is known. Having thus determined the function $y(\xi)$, one can obtain from relations (42), (43), (39a) the functions $\beta(\xi)$, $\varphi(\xi)$ and $f(\xi)$ and complete the solution of this problem. Thus, in order that the problem may be solved in a closed form, it is necessary to determine the initial condition $f(0)$ or the mass transfer parameter B_D.

From relations (36) and (42) determine $\psi'(0)$ and $\beta'(0)$ and substitute these into boundary condition (31).

Taking into account relation (34), one obtains

$$\frac{\lambda_f}{(\rho D)_W}\frac{(T_W-T_0)}{(1+B_D)}\cdot\frac{\rho_e R T_W}{m_N p_e} \times\left[\sum_{i=2}^{N}\frac{m_N}{m_i}(Y_{ie}-\Phi_i Y_{1e})+B_D\right] \times\exp\left[\frac{f^2(0)}{2}\left(\frac{1}{a^2}-\frac{\rho_e^2}{\rho_f^2}\frac{1}{b^2}\right)\right] = \frac{1-\operatorname{erf}\{[\rho_e/\rho_f][f(0)/b\sqrt{2}]\}}{1+\operatorname{erf}[f(0)/b\sqrt{2}]} \times[\hat{c}_{pe}(T_e-T_W)+Y_{1e}Q_{pW}-B_D h_L]. \tag{45}$$

Equations (40) and (34) yield

$$\rho_e f(0)=\{\sqrt{2}\cdot e^{-f^2(0)/2a^2}(\rho D)_W B_D(1+B_D)p_e m_N\} \div\left\{a\sqrt{\pi}\cdot\{1+\operatorname{erf}[f(0)/a\sqrt{2}]\} \times\left[\sum_{i=2}^{N}\frac{m_N}{m_i}(Y_{ie}-Y_{1e}\Phi_i)+B_D\right]\rho_e R T_W\right\}. \tag{46}$$

Relations (45), (46) and (33) represent a closed system of equations to determine the unknown quantities $f(0)$, T_W and B_D.

The position of the diffusion flame surface in the case of the known solution $\beta(\xi)$ is determined from the condition that fuel vapours are absent above the surface ($Y_N = 0$) and oxidant vapours are absent under the flame surface ($Y_1 = 0$). Then, for the pair function β_N there is the same, as in refs. [7, 8, 14], condition

$$\beta_N(\xi_*) = 0,$$

whence, using the first integrals of equation (18) and the conditions at the phase interface (30), the following relation is obtained to determine the coordinates of the flame surface ξ_*:

$$\beta(\xi_*)=\frac{Y_{NW}}{Y_{NW}+Y_{1e}\Phi_N}=\frac{B_D-Y_{1e}\Phi_N}{B_D(1+Y_{1e}\Phi_N)}. \tag{47}$$

Let us consider one of the possible approximations [10]; namely, assume that $T_W = T_{ke}$, where T_{ke} is the equilibrium boiling temperature at the pressure p_e. In this case, it is seen from relation (33) that $Y_{ie} - Y_{1e}\Phi_i = 0$ for all $i = 2,\ldots,N-1$, which is equivalent to the condition $Y_{iW} = 0\ \forall i = 1,\ldots,N-1$; $Y_{NW} = 1$. Relation (45) is simplified and it can be used to determine the mass transfer parameter B_D

$$B_D=\frac{Y_{1e}Q_{pW}}{h_L}+\frac{\hat{c}_{pe}(T_e-T_{ke})}{h_L}-\frac{\lambda_f}{(\rho D)_W}\cdot\frac{\rho_e R T_{ke}}{m_N p_e} \times\frac{(T_{ke}-T_0)\{1+\operatorname{erf}[f(0)/a\sqrt{2}]\}}{1-\operatorname{erf}\{[\rho_e/\rho_f][f(0)/b\sqrt{2}]\}} \times\exp\left[\frac{f^2(0)}{2}\left(\frac{1}{a^2}-\frac{\rho_e^2}{\rho_f^2 b^2}\right)\right]. \tag{48}$$

The substitution of the above relation into relation (46) yields the transcendental equation to determine the unknown boundary condition $f(0)$:

$$\frac{a\sqrt{\pi}}{\sqrt{2}}\rho_e f(0)=\frac{p_e m_N(\rho D)_W[Y_{1e}Q_{pW}+\hat{c}_{pe}(T_e-T_{ke})]}{\rho_e R T_{ke} h_L\left(1+\operatorname{erf}\dfrac{f(0)}{a\sqrt{2}}\right)\exp\left(\dfrac{f^2(0)}{2a^2}\right)} -\lambda_f\frac{(T_{ke}-T_0)}{1-\operatorname{erf}\{[\rho_e/\rho_f][f(0)/b\sqrt{2}]\}} \times\exp\{-[\rho_e f(0)/\rho_f b\sqrt{2}]^2\}. \tag{49}$$

4. ANALYSIS OF RESULTS

In the solution of many problems of practical interest the main concern is not the distribution of parameters above the combustible surface, but rather the mass rate of the surface burn out. The solution obtained completely meets the case. With the known value of $f(0)$, determined, for instance, from the solution of system (45), (46), (33) or roughly from the solution of relation (49), the mass rate of fuel burn-out is prescribed by the relation

$$(\rho v)_W = \rho_e f(0)/\sqrt{2t}. \tag{50}$$

Thus, the proposed method for the determination of the surface burn-out rate allows one to integrate the system of differential equations and to reduce the solution of the problem to the solution of algebraic and transcendental equations.

A change in the level of the combustible surface as it burns out can be described by the formula obtained by carrying out the integration of relation (50):

$$X_W(t)=\int_0^t v_f(t)\,dt=\frac{\rho_e}{\rho_f}f(0)\sqrt{2t}. \tag{51}$$

Experiments on one-dimensional combustion of liquids in fine elongated burners with the diameters from 6 to 23 mm were conducted by Blinov and Khudyakov [2]. Ignition was initiated near the liquid surface and as the time proceeded a change in the liquid level in a burner was determined. The analysis of the experimental data has shown that a change in X_W can be satisfactorily described by the relation

$$X_W = Kt^n,$$

where the exponent n varies (for different tests) within the range 0.55–0.75. Thus, formula (51) agrees satisfactorily with the experimental data. Experimental investigations of the combustion of diesel fuels [2] showed that the rate of surface burning v_W, equation (50), decreases with the lapse of time and, on reaching the critical value v_W^*, combustion terminates, i.e. the self-quenching of flame takes place (for diesel fuels $v_W^* = 0.41 \times 10^{-5}$ m s^{-1}).

The dependence of the combustion rate on the fuel surface level X_W can be easily determined from formulae (50) and (51):

$$v_W=\left[\frac{\rho_e}{\rho_f}f(0)\right]^2\frac{1}{X_W}. \tag{52}$$

Table 1. Diffusion combustion of gasoline

Parameter	T_e	T_0	T_W	Y_{1e}	Y_{NW}	ρ_e	ρ_f	$f(0)$
Dimensionality	K	K	K	—	—	kg m^{-3}	kg m^{-3}	m s$^{-1/2}$
Numerical value	300	300	356	0.30	0.89	1.25	740	0.19

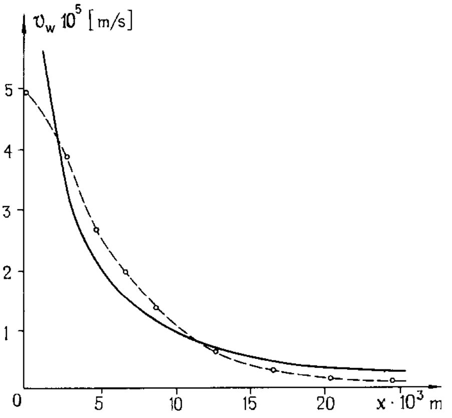

FIG. 2. Variation in the burn-out rate with liquid surface receding downward (solid curve, theoretical calculation; dashed curve, experiment).

The results of calculation of the main governing parameters for the problem of unsteady-state combustion of gasoline surface, contacting with air, are given in Table 1.

Figure 2 presents the dependence of the gasoline combustion rate on the surface level plotted on the basis of formula (52) and the data of Table 1 (solid curve). For comparison, the dashed curve gives the results of experimental investigations [2].

Thus, an exact solution to the similarity problem of diffusion combustion of a fuel surface has been obtained in the paper. The results of calculations agree well with the experimental data.

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TRANSFERT DE CHALEUR ET DE MASSE PENDANT LA COMBUSTION DE DIFFUSION VARIABLE D'UNE SURFACE DE COMBUSTIBLE CONDENSEE

Résumé—On obtient une solution du problème variable unidimensionnel de la combustion dans une atmosphère oxydante d'une surface de combustible condensée. Comme dans la majorité des problèmes de combustion de diffusion, la vitesse de la réaction chimique est supposée grande comparée à la vitesse de diffusion. Une solution analytique est obtenue pour le système d'équations monodimensionnelles et variables qui décrit l'écoulement d'un mélange en réaction de gaz conducteurs de la chaleur et diffusants. On fait une comparaison entre les résultats de calcul et les données expérimentales sur la combustion de diffusion d'essence dans des tubes fins.

WÄRME- UND STOFFÜBERTRAGUNG BEI INSTATIONÄRER DIFFUSIONSVERBRENNUNG AN DER OBERFLÄCHE EINES FLÜSSIGEN BRENNSTOFFES

Zusammenfassung—Es wird eine Lösung für das instationäre eindimensionale Problem der Verbrennung an der Oberfläche eines flüssigen Brennstoffes in einer oxidierenden Atmosphäre ermittelt. Wie bei der Mehrzahl der Diffusionsverbrennungsprobleme wird die Geschwindigkeit der chemischen Reaktion als groß im Vergleich zur Diffusionsgeschwindigkeit angenommen. Eine analytische Lösung für das System von instationären eindimensionalen Gleichungen, welche die Strömung einer wärmeleitenden reagierenden Mischung aus wechselseitig diffundierenden Gasen beschreiben, wird ermittelt. Außerdem werden die Ergebnisse der Berechnung mit experimentellen Daten der Diffusionsverbrennung von Benzin in dünnen Rohren verglichen.

ТЕПЛОМАССООБМЕН ПРИ НЕСТАЦИОНАРНОМ ГОРЕНИИ ПОВЕРХНОСТИ КОНДЕНСИРОВАННОГО ТОПЛИВА В ДИФФУЗИОННОМ РЕЖИМЕ

Аннотация—Решена нестационарная одномерная задача о горении поверхности конденсированного горючего в атмосфере окислителя. Как и в большинстве задач диффузионного горения предполагается, что в зоне пламени скорость химической реакции велика по сравнению со скоростью диффузии. Получено аналитическое решение системы нестационарных одномерных уравнений, описывающих течение теплопроводной, химически реагирующей смеси взаимно диффундирующих газов. Проводится сравнение результатов проведенных расчетов с экспериментальными данными по диффузионному горению бензина в тонких трубах.